Comment on "Classical and Quantum Interaction of the Dipole"

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In [1], Anandan has presented a covariant treatment of the interaction of the electric and magnetic dipole moments of a particle with the electromagnetic field. Our aim is to make some important changes of the results from [1]. Instead of dealing with component form of tensors E^{μ}, .. [1], we shall deal with tensors as four-dimensional (4D) geometric quantities, E^{a}, .. . For simplicity, only the standard basis $\{e_{\mu};\ 0,1,2,3\}$ of orthonormal 4-vectors, with e_0 in the forward light cone, will be used.

Anandan states: "In any frame D^{0i} and D^{ij} that couple, respectively, to the electric field components F_{0i} and the magnetic field components F_{ij} are called the components of the electric and magnetic dipole moments." Then, he defines that d^{μ} and m^{μ} are the components of $D^{\mu\nu}$, Eq. (2), and similarly that E^{μ} and B^{μ} are the components of $F^{\mu\nu}$, Eq. (4). Several objections can be raised to such treatment.

It is proved in [2] that the primary quantity for the whole electromagnetism is F^{ab} (i.e., in [2], the bivector F). F^{ab} can be decomposed as $F^{ab}=(1/c)(E^{a}v^{b}-E^{b}v^{a})+\varepsilon^{abcd}v_{c}B_{d}$, whence $E^{a}=(1/c)F^{ab}v_{b}$ and $B^{a}=(1/2c^{2})\varepsilon^{abcd}F_{bc}v_{d}$, with $E^{a}v_{a}=B^{a}v_{a}=0$; only three components of E^{a} and B^{a} in any basis are independent. The 4-velocity v^{a} is interpreted as the velocity of a family of observers who measure E^{a} and B^{a} fields. E^{a} and B^{a} depend not only on F^{ab} but on v^{a} as well. In the frame of "fiducial" observers, in which the observers who measure E^{a}, B^{a} are at rest, $v^{a}=ce_{0}$. That frame with the $\{e_{\mu}\}$ basis will be called the e_0-frame. In the e_0-frame $E^{0}=B^{0}=0$ and $E^{i}=F^{i0}$, $B^{i}=(1/2c)\varepsilon^{ijk0}F_{jk}$. In any other inertial frame, the "fiducial" observers are moving, and $v^{a}=ce_{0}=v^{\prime\mu}e_{\mu}^{\prime}$; under the passive Lorentz transformations (LT), $v^{\mu}e_{\mu}$ transforms as any other 4-vector transforms. The same holds for E^{a} and B^{a}, e.g., $E^{a}=E^{\mu}e_{\mu}=[(1/c)F^{i0}v_{0}]e_{i}=E^{\prime\mu}e_{\mu}^{\prime}=[(1/c)F^{\prime\mu\nu}v_{\nu}^{\prime}]e_{\mu}^{\prime}$. E^{μ} transform by the LT again to the components $E^{\prime\mu}$ of the same electric field. There is no mixing with the components of the magnetic field. $E^{\prime\mu}$ are not determined only by $F^{\prime\mu\nu}$ but also by $v^{\prime\mu}$.

Only in the e_0-frame, and thus not in any frame, are F^{i0} and F^{jk} the electric and magnetic field components, respectively. The assertion that, e.g., in any inertial frame it holds that $E^{0}=E^{\prime 0}=0$, $E^{i}=F^{i0}$, and $E^{\prime i}=F^{\prime i0}$, leads to the usual transformations of the 3-vector $\mathbf{E}$, see, e.g., [3], Eq. (11.149). In [4], the fundamental results are achieved that these usual transformations of the 3-vectors $\mathbf{E}$ and $\mathbf{B}$ are not relativistically correct and have to be replaced by the LT of the electric and magnetic fields as 4D geometric quantities.

The electric and magnetic dipole moment 4-vectors d^{a} and m^{a}, respectively, can be determined from dipole moment tensor D^{ab} in the same way as E^{a} and B^{a} are obtained from F^{ab}; $D^{ab}=(1/c)(u^{a}d^{b}-u^{b}d^{a})+(1/c^{2})\varepsilon^{abcd}u_{c}m_{d}$, whence $d^{a}=(1/c)D^{ba}u_{b}$, and $m^{a}=(1/2)\varepsilon^{abcd}D_{bc}u_{d}$, with $d^{a}u_{a}=m^{a}u_{a}=0$. $u^{a}=dx^{a}/ds$ is the 4-velocity of the particle.

The whole discussion about E^{a}, B^{a} and F^{ab} can be repeated for d^{a}, m^{a} and D^{ab}. Now, only in the rest frame of the particle and the $\{e_{\mu}\}$ basis, $u^{a}=ce_{0}$ and $d^{0}=m^{0}=0$, $d^{i}=D^{0i}$, $m^{i}=(c/2)\varepsilon^{ijk0}D_{jk}$.

It is also stated in [1]: "The electric and magnetic fields in the rest frame ... ." But, there is no rest frame for fields. The whole discussion in [1] has to be changed using different 4-velocities v^{a} and u^{a}. Thus Eqs. (7) and (6) become $(1/2)F_{ab}D^{ba}=(1/c)D_{a}u^{a}+(1/c^{2})M_{a}u^{a}$, and $D_{a}=d^{b}F_{ba}$, $M_{a}=m^{b}F_{ba}^{*}$. Instead of Eq. (5), we have that $(1/2)F_{ab}D^{ba}$ is the sum of two terms $(1/c^{2})[((E_{a}d^{a})+(B_{a}m^{a}))(v_{b}u^{b})-(E_{a}u^{a})(v_{b}d^{b})-(B_{a}u^{a})(v_{b}m^{b})]$ and $(1/c^{3})[\varepsilon^{abcd}(v_{a}E_{b}u_{c}m_{d}+c^{2}d_{a}u_{b}v_{c}B_{d})]$; the second term contains the interaction of E_{a} with m^{a}, and B_{a} with d^{a}. This last result significantly influences Eq. (17), and it will give new interpretations for, e.g., the Aharonov-Casher and the Röntgen phase shifts.

References

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